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Bernoulli's Equation

1 $3\frac{dy}{dx}+\frac{2}{x+1}y=\frac{x^3}{y^2}$

$$3y^2\frac{dy}{dx}+\frac{2}{\cancel{3}}\frac{1}{x+1}y\cdot y^2=x^3$$

$$3y^2\frac{dy}{dx}+\frac{2}{\cancel{3}}\frac{y^3}{x+1}=x^3$$

Put $y^3=z$

$\Rightarrow 3y^2\,dy=dz$

$$\frac{dz}{dx}+\frac{2z}{x+1}=x^3$$

now equation linear in z

I.F $e^{\int\left(\frac{2}{x+1}\right)dx}$

$$=e^{2\log(x+1)}=e^{\log(x+1)^2}=(x+1)^2$$

Solution is

$$z\cdot(x+1)^2=\int x^3(x+1)^2\,dx+c$$

$$z\cdot(x+1)^2=\int x^3(x^2+1+2x)+c$$

$$=\int(x^5+2x^4+x^3)\,dx+c$$

$$z\cdot(x+1)^2=\frac{x^6}{6}+\frac{2x^5}{5}+\frac{x^4}{4}+c$$

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2. $(y\log x - 1)y\,dx = x\,dy$

Sol. $x\frac{dy}{dx} = (y\log x - 1)y$

$x\frac{dy}{dx} = y^2\log x - y$

$x\frac{dy}{dx} + y = y^2\log x$

Divide the whole equation by y^2

$\frac{x}{y^2}\frac{dy}{dx} + \frac{y}{y^2} = \log x$

$xy^{-2}\frac{dy}{dx} + y^{-1} = \log x$

Put $y^{-1} = z$ or $\frac{1}{y} = z$

$-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$ or $\frac{1}{y^2}\frac{dy}{dx} = -\frac{dz}{dx}$

$-x\frac{dz}{dx} + z = \log x$

$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x}\log x$

now, it is linear differential equation in z

I.F $e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Solution is

$z\cdot\frac{1}{x} = \int \frac{1}{x}\log x \cdot \frac{-1}{x}\,dx + c$

$$z \cdot \frac{1}{x} = -\int \log x \frac{1}{x^2} dx + c$$

Integraing in parts, taking $\log x$ as first function

$$z \cdot \frac{1}{x} = -\left[\log x \left(-\frac{1}{x}\right) - \int \frac{1}{x}\left(-\frac{1}{x}\right) dx \right] + c$$

$$z \cdot \frac{1}{x} = \frac{1}{x} \log x + \frac{1}{x} + c$$

$$z = \log x + 1 + cx$$

$$z = \frac{1}{y} = \log x + 1 + cx$$

3. $(x-y^2)\,dx + 2xy\,dy = 0$

Re-write the given equation

$$2xy\,dy = -(x-y^2)\,dx$$

$$2xy\frac{dy}{dx} = -(x-y^2)$$

$$2xy\frac{dy}{dx} = -x+y^2$$

$$2xy\frac{dy}{dx} - y^2 = -x$$

$$2y\frac{dy}{dx} - \frac{y^2}{x} = -\frac{x}{x}$$

$$2y\frac{dy}{dx} - \frac{y^2}{x} = -1$$

Put $y^2 = z$ $\quad 2y\frac{dy}{dx} = \frac{dz}{dx}$

$$\frac{dz}{dx} - \frac{z}{x} = -1$$

I.f $e^{-\int\frac{1}{x}dx} = e^{-\log x}$

$$= e^{\log x^{-1}} = \frac{1}{x}$$

Solution is

$$z\cdot\frac{1}{x} = \int\frac{1}{x}\cdot 1\,dx + c$$

$z\frac{1}{x} = \log x + c$

$y^2 = x(\log x + c)$

1. $\sin y \frac{dy}{dx} = \cos y(1 - x\cos y)$

Solution:

$$\frac{\sin y}{\cos y}\frac{dy}{dx} = 1 - x\cos y$$

Divide whole equation by $\cos y$

$$\frac{\sin y}{\cos^2 y}\frac{dy}{dx} - \frac{1}{\cos y} = -x$$

Put $\frac{-1}{\cos y} = z$

$$\frac{1}{\cos^2 y}(-\sin y)\frac{dy}{dx} = \frac{-dz}{dx}$$

$$-\frac{dz}{dx} + z = -x \quad \text{or} \quad \frac{dz}{dx} - z = x$$

I.F $e^{-\int dx} = e^{-x}$

Solution is

$$z \cdot e^{-x} = \int + x \cdot e^{-x} + c$$

$$z \cdot e^{-x} = -xe^{-x} - \int 1 \cdot (-e^{-x})\, dx + c$$

$$z e^{-x} = -xe^{-x} - e^{-x} + c$$

$$z = -x - 1 + c$$

$$\frac{-1}{\cos y} = -x - 1 + c$$

$$\sec y = x + 1 - c$$

5. $\left(xy - e^{1/x^3}\right) dx - x^2 y\, dy = 0$

Solution:- $\left(xy - e^{1/x^3}\right) dx = x^2 y\, dy$

$$x^2 y \frac{dy}{dx} = \left(xy^2 - e^{1/x^3}\right) dx$$

Divide whole equation by x^2

$$y \frac{dy}{dx} = \frac{y^2}{x} - \frac{e^{1/x^3}}{x^2}$$

$$y \frac{dy}{dx} - \frac{y^2}{x} = -\frac{e^{1/x^3}}{x^2}$$

Put $y^2 = z$

$$2y \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y \frac{dy}{dx} = \frac{1}{2} \frac{dz}{dx}$$

$$\frac{1}{2}\frac{dz}{dx} - \frac{z}{x} = -\frac{e^{1/x^3}}{x^2}$$

$$\frac{dz}{dx} - \frac{2}{x}z = -\frac{2}{x^2}e^{1/x^3}$$

I.F $\quad e^{-2\int\frac{1}{x}dx}$

$$= e^{-2\log x}$$

$$= e^{\log x^{-2}} = \frac{1}{x^2}$$

Solution of linear differential Equation

$$z\cdot\frac{1}{x^2} = \int \frac{1}{x^2} \cdot -\frac{2}{x^2} e^{1/x^3}\, dx + c$$

$$z\cdot\frac{1}{x^2} = -2\int \frac{1}{x^4} e^{1/x^3}\, dx + c$$

Put $\frac{1}{x^3} = t$

$$-\frac{3}{x^4}dx = dt$$

$$\Rightarrow \frac{dx}{x^4} = -\frac{1}{3}dt$$

$$z\cdot\frac{1}{x^2} = \frac{-2}{-3}\int e^t\, dt + c$$

$$3z\cdot\frac{1}{x^2} = 2e^t + c$$

$$3y^2 = 2x^2 e^{1/x^3} + cx^2$$